Anisotropic pair-superfluidity of trapped two-component Bose gases

Yongqiang Li[1,2], Liang He[1] and Walter Hofstetter[1]
[1] *Institut für Theoretische Physik, Goethe-Universität, 60438 Frankfurt am Main, Germany*
[2] *Department of Physics, National University of Defense Technology, Changsha 410073, P. R. China*
(Dated: March 21, 2012)

We theoretically investigate the pair-superfluid phase of two-component ultracold gases with negative inter-species interactions in an optical lattice. We establish the phase diagram for filling $n = 1$ at zero and finite temperature, by applying Bosonic Dynamical Mean-Field Theory, and confirm the stability of pair-superfluidity for asymmetric hopping of the two species. While the pair superfluid is found to be robust in the presence of a harmonic trap, we observe that it is destroyed already by a small population imbalance of the two species.

PACS numbers: 03.75.Kk, 03.75.Mn, 67.85.-d, 67.85.Hj

I. INTRODUCTION

Ultracold gases in an optical lattice are promising as flexible quantum simulators for the study of quantum phases which are not easily accessible in condensed-matter physics [1]. Experimental realization of the superfluid-to-Mott transition has paved the way to studies of strongly interacting Bose or Fermi gases in optical lattices [2]. Recently, Bose-Bose mixtures of ^{87}Rb and ^{41}K have been produced and loaded into an optical lattice [3], which provide a new playground for investigating the interplay between kinetic energy, interaction and the spin degree of freedom. In a further line of investigation, a Bose-Bose mixture of two hyperfine states of ^{87}Rb has been used as a spin-gradient thermometer for measuring the temperature of ultracold gases in an optical lattice [4, 5]. Moreover, the effect of a second species on bosonic superfluidity in an optical lattice has been studied [6]. In these mixtures of different species or different hyperfine states, the most fundamental physics associated with quantum magnetism and the spin degree of freedom can be explored [7].

Recently, it was found that a three-body hard-core constraint can stabilize a system of single-component bosons with attractive two-body interactions in an optical lattice [8], and numerical simulations have been performed to study properties of this system [9–11]. For two-component bosons with attractive inter-species interactions in an optical lattice, one interesting ground state is the pair-superfluid phase (PSF), which has been predicted and studied theoretically in several papers [12–19]. Qualitatively, the PSF can be viewed as a condensate of bosonic pairs of different species or hyperfine states due to a second-order hopping of the pairs, but with a strongly suppressed first-order tunneling of single atoms. For hard-core bosons with total filling $n = 1$, the PSF for attractive interactions is equivalent to the XY-ferromagnetic phase (or super-counter-fluid state, denoted as SCF) for repulsive interactions, i.e. the PSF consists of particle-particle pairs of different species while the XY-ferromagnetic phase is composed of particle-hole pairs. It is apparent that the PSF can only exist at very low temperature compared to the critical temperature of quantum magnetic phases. The latter are also governed by second-order tunneling processes, leading to an effective spin-exchange coupling [20, 21], which has already been observed for a double-well system [22, 23]. At the current stage, it is experimentally challenging to reach the critical temperatures of quantum magnetic phases and the PSF [5]. It is expected that these phases can be detected via momentum-space correlations observed in time-of-flight measurements [17, 24] or by optical microscopy with single-site resolution [25, 26].

Previous studies of the PSF in a two- or three-dimensional optical lattice mostly focus on symmetric parameters for the two species (for an exception in [12, 18]), since this is the most favorable condition for the PSF [17]. However, there is still a lack of detailed quantitative investigations of the PSF for the homogeneous system with asymmetric hopping amplitudes of the two species, for the trapped system, and for imbalanced mixtures of the two species. Here we investigate the properties of the PSF of two-component ultracold bosons with attractive interspecies interaction, both in a homogeneous and a trapped optical lattice. This system can for sufficiently low filling be well described by a single-band Bose-Hubbard model with pure onsite interaction. We investigate the homogeneous system by means of bosonic dynamical mean field theory (BDMFT), which is a non-perturbative approach towards strongly-correlated bosonic systems [27], and the trapped system by real-space bosonic dynamical mean field theory (RBDMFT) [28], which includes arbitrary inhomogeneity such as a harmonic trap. For the homogeneous system, we focus on the phase diagram with filling number $n = 1$. In particular, we also present the phase diagram for the experimentally realized heteronuclear ^{87}Rb - ^{41}K mixtures, where double-species Bose-Einstein condensates with negative inter-species interactions have been observed [29]. For the trapped Bose-Bose mixture we study the coexistence of Mott insulator, superfluid and PSF.

The paper is organized as follows: in section II we give a detailed description of the Bose-Hubbard model and the BDMFT approach. Section III covers our results on the homogeneous Bose-Bose mixture in an optical lattice and the effect of the trap. We conclude in Section V.

II. MODEL AND METHOD

We consider a two-component bosonic mixture loaded into a 2D or 3D optical lattice. In experiments this mixture could consist of two different species, e.g ^{87}Rb and ^{41}K as in [3] or two different hyperfine states of a single isotope, e.g ^{87}Rb as in [4]. Besides the optical lattice, we also impose an external harmonic trapping potential which introduces inhomogeneity in the system. For sufficiently low filling, the whole system can be effectively described by a two-component inhomogeneous Bose-Hubbard model within the single-band approximation

$$\mathcal{H} = -\sum_{\substack{<i,j>\\ \nu=b,d}} t_\nu (b^\dagger_{i\nu} b_{j\nu} + h.c) + \frac{1}{2}\sum_{i,\lambda\nu} U_{\lambda\nu}\hat{n}_{i,\lambda}(\hat{n}_{i\lambda} - \delta_{\lambda\nu}) + \sum_{i,\nu=b,d}(V_i - \mu_\nu)\hat{n}_{i\nu} \quad (1)$$

where $\langle i,j\rangle$ denotes summation over nearest neighbor sites i,j, and the two bosonic species are labeled by the index $\lambda(\nu) = b,d$. $\hat{b}^\dagger_{i\nu}$ ($\hat{b}_{i\nu}$) denotes the bosonic creation (annihilation) operator for species ν at site i and $\hat{n}_{i,\nu} = \hat{b}^\dagger_{i\nu}\hat{b}_{i\nu}$ the corresponding local density. Due to possibly different masses or a spin-dependent optical lattice, these two species generally hop with non-equal amplitudes t_b and t_d. $U_{\lambda\nu}$ denotes the inter- and intra-species interactions, which can be tuned via Feshbach resonances or spin-dependent lattices [6, 30]. μ_ν is the global chemical potential for the two bosonic species, and $V_i \equiv V_0\, r_i^2$ denotes the external harmonic trap, where V_0 is the strength of the harmonic trap and r_i is the distance from the trap center.

To investigate properties of the system, we apply BDMFT for a homogeneous lattice [27], and its real-space generalization (RBDMFT) for the trapped system [28]. Within RBDMFT, the Hamiltonian (1) is mapped onto a set of individual single-site problems, and the physical properties on each lattice site are determined by a local effective action which is captured by an effective Anderson impurity model [27] solved by exact diagonalization (ED) [27, 31]. RBDMFT fully captures the inhomogeneity of the system and is capable of providing an accurate and non-perturbative description of quantum phases and their excitations. The detailed formalism is presented in [28].

To include the effect of spatial inhomogeneity, we also employ an LDA approximation combined with single-site BDMFT. The advantage of this approach is the larger system size accessible. Within LDA+BDMFT, the local chemical potential for each species is set to $\mu_\nu(r) = \mu_\nu - V(r)$. In this work, we apply both RBDMFT and LDA+BDMFT to the 2D trapped square lattice, and only LDA+BDMFT to the 3D trapped cubic lattice.

III. RESULTS

In this section, we will investigate properties of two-component bosonic mixtures with negative inter-species interactions in a homogeneous 3D optical lattice at zero and finite temperature, and also in harmonically trapped, inhomogeneous systems both in 2D and 3D. For the homogeneous system, we will study the stability of the PSF against asymmetric hopping and finite temperature. In the presence of an external harmonic trap, we will investigate finite size effects of the PSF both in 2D and 3D within RBDMFT and LDA+BDMFT. We choose the absolute value of the inter-species interaction $U \equiv U_{bd}$ as our unit of energy in the following.

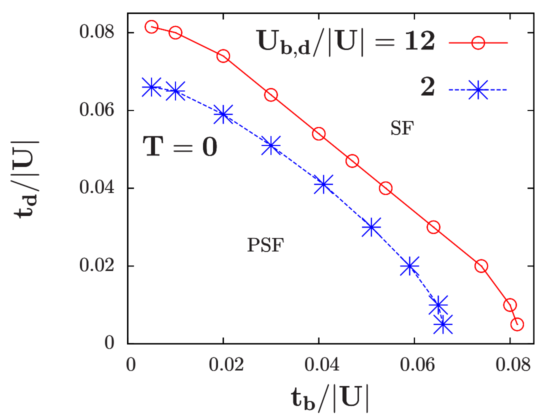

FIG. 1: Zero-temperature phase diagram for two-component bosons with attractive inter-species interaction in a 3D cubic lattice. The total filling is $n = 1$ with $n_b = n_d = 0.5$.

A. Bose-Bose mixture on a 3D cubic lattice

We first investigate Bose-Bose mixtures with negative inter-species interaction $U < 0$ in a 3D cubic optical lattice. The system is unstable for $|U| > U_{b,d}$, since the strongly attractive inter-species interaction cannot be compensated by repulsive intra-species interactions, leading to a collapse of the system. Here we will demonstrate the stability of the PSF in the regime $|U| < U_{b,d}$. We explore the zero-temperature phase diagram with asymmetric hopping parameters at total filling $n = 1$ (with $n_b = n_d = 0.5$) for different interactions, as shown in Fig. 1. We observe two different phases: the PSF with $\langle b\rangle = \langle d\rangle = 0$ but $\phi_{bd} \equiv \langle bd\rangle - \langle b\rangle\langle d\rangle \neq 0$, and the superfluid phase (SF) with $\langle b\rangle, \langle d\rangle > 0$. In particular, we confirm the existence of a PSF for asymmetric hopping amplitudes $t_b \neq t_d$. In the regime of weak hopping for both species, first-order tunneling is suppressed by the strong interactions. Formation of bosonic pairs between different species can thus be energetically favored and will typically compete with single-species condensation, since the bosonic pairs can hop via second-order tunneling. As a result, the PSF will have a non-vanishing order

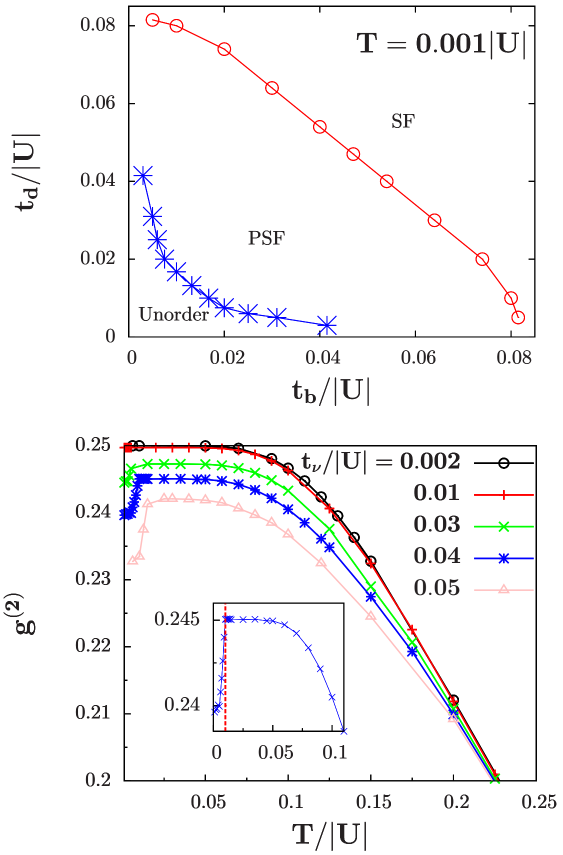

FIG. 2: **Upper**: Finite-temperature phase diagram for two-component bosons in a 3D cubic lattice ($T = 0.001|U|$). The interactions are set to $U_b = U_d = 12|U|$ and the total filling is $n = 1$ with $n_b = n_d = 0.5$. **Lower**: Local density-density correlator $g^{(2)}$ as a function of temperature for different hopping values $t_b = t_d$. Inset: Zoom of main figure for $t_\nu = 0.04|U|$ ($\nu = b, d$), where the red dashed line shows the disappearance of the PSF correlator ϕ_{bd}.

parameter ϕ_{bd} but vanishing superfluid order $\langle b\rangle$ and $\langle d\rangle$. On the other hand, when both species acquire large hopping, a superfluid phase will appear with $\langle b\rangle > 0$ and $\langle d\rangle > 0$. Note that, when the intra-species interaction $U_{b,d}$ decreases from $U_b = U_d = 12|U|$ to $2|U|$, the PSF shrinks due to the decrease of the effective pair-tunneling amplitudes. We remark here that the transition from a PSF to a superfluid phase for symmetric hopping amplitudes occurs at the same value of t_ν/U_{bd} as that from the XY-ferromagnetic to the superfluid phase in the corresponding system with $U_{bd} > 0$ of equal magnitude. In contrast to the XY-ferromagnetic phase, however, the PSF also exists for non-integer filling.

The effect of finite temperature is shown in Fig. 2. Generally, the PSF is sensitive to temperature, since the pairs are formed in the weak hopping regime and their coherence can be easily destroyed by thermal fluctuations, due to their small effective tunneling of order $O(t^2/U)$. At finite temperature, the PSF regime shrinks in the weak hopping regime in favor of developing a new unordered phase with vanishing values for ϕ_{bd}, $\langle b\rangle$ and $\langle d\rangle$. To further understand this unordered phase, we calculate the dependence of the local density-density correlator $g^{(2)} \equiv \langle n_b n_d\rangle - \langle n_b\rangle\langle n_d\rangle$ on temperature, as shown in the lower panel of Fig. 2. We observe that $g^{(2)}$ starts to decrease noticeably only above temperatures of the order of $10^{-1}|U|$, which indicates that local pairs still exist below this temperature. We therefore conclude that the unordered phase, shown in the upper panel of Fig. 2, consists of non-coherent pairs of different species. In sufficiently deep optical lattices, these pairs are localized, while for larger hopping the pairs delocalize over the whole lattice. As a result, the local density-density correlator decreases as a function of $t_{b,d}$, as shown in the lower panel of Fig. 2. Another interesting feature of the temperature dependence of $g^{(2)}$ is the increasing (non-monotonic) trend at low temperatures, since thermal fluctuations first localize and then break the pairs. We remark here that the temperature regime of non-condensed pairs ($\approx 0.1|U|$) is experimentally accessible [32], and could be detected via radio frequency spectroscopy [33]. Note that the unordered phase observed here is qualitatively different from that melted from the XY-ferromagnetic phase, since it can also exist for non-integer filling. We also observe that the (single-particle) superfluid phase remains almost unchanged for the temperature considered here.

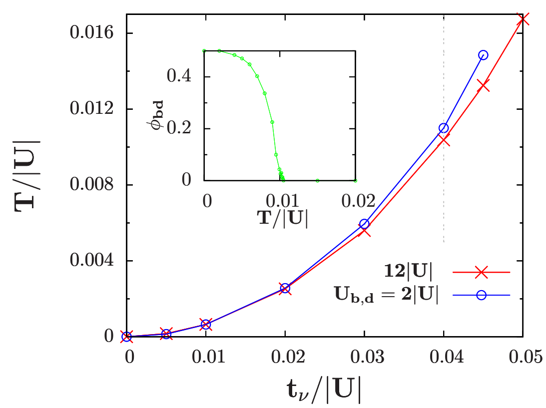

FIG. 3: Critical temperature of the PSF as a function of hopping amplitudes $t_b = t_d$ on a 3D cubic lattice with total filling $n = 1$. Inset: melting of the PSF vs. temperature along the vertical dashed line with hopping amplitudes $t_b = t_d = 0.04\,|U|$.

One crucial question regarding the observation of the PSF is how fragile it is against finite-temperature effects. To address this issue, Fig. 3 shows T_c as a function of the hopping amplitudes $t_b = t_d$ at different interactions. We notice that T_c rises as the hopping amplitudes increase, due to the growing second-order tunneling which stabilizes long-range order. The inset of Fig. 3 shows the temperature dependence of ϕ_{bd}. It indicates a second-order phase transition from the PSF to the unordered phase. We also observe that the critical temperatures for the PSF shown here are comparable to those of the XY-ferromagnetic phase [28] and notably smaller than the coldest temperatures which have been measured in most experiments until now, with the exception of the MIT group where temperatures as low as 350pK ($\approx 0.01 U_{bd}$ with $t_b/U_{bd} \approx 0.029$) have been achieved [5].

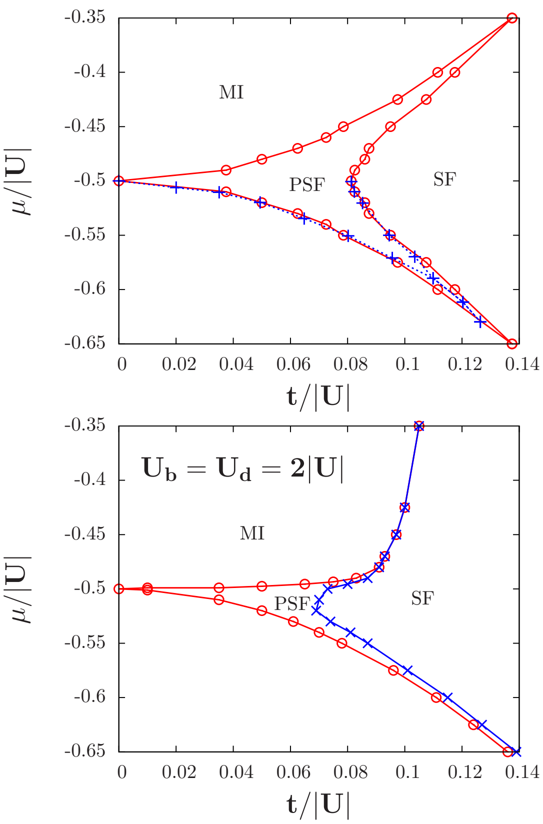

FIG. 4: **Upper**: Comparison of the zero-temperature phase diagram for two-component hard-core bosons with the one obtained by a tensor-product-state approximation [19] for a square lattice with symmetric parameters: $t = t_b = t_d$, $\mu = \mu_b = \mu_d$. The red solid lines are phase boundaries obtained by BDMFT for $U_b = U_d = 200|U|$, while the blue dashed lines are the results of the tensor-product-state approximation. **Lower**: Zero-temperature phase diagram for two-component soft-core bosons with $U_b = U_d = 2|U|$ obtained via BDMFT.

To verify the validity of the BDMFT results, comparison has been made with a hard-core boson model solved by a tensor-product-state approximation [19], as shown in Fig. 4. We find excellent agreement between the two methods. We also plot the phase diagram for soft-core bosons ($U_b = U_d = 2|U|$), and observe that in this case the phase boundary between the PSF and the superfluid phase is shifted to lower hopping values.

B. Rubidium-potassium mixture

Our investigations have so far focused on symmetric interactions $U_b = U_d$, which is a good approximation for mixtures of hyperfine states of ^{87}Rb [4]. However, this symmetry is not present for mixtures of ^{87}Rb and ^{40}K, where a negative inter-species interaction has been achieved via a Feshbach resonance [29]. Here we consider a mixture of ^{87}Rb and ^{41}K loaded into a 3D cubic lattice with wavelength $\lambda = 757$ nm, which yields equal dimensionless lattice strength s for the two species. Due to the different masses, the ratio of the intra-species interaction strengths is then fixed to $U_{\mathrm{Rb}}/U_{\mathrm{K}} = m_{\mathrm{K}} a_{\mathrm{Rb}}/m_{\mathrm{Rb}} a_{\mathrm{K}} \approx 0.72$ and the ratio of the hopping amplitudes to $t_{\mathrm{Rb}}/t_{\mathrm{K}} \approx 0.47$, where E_{R} is the recoil energy.

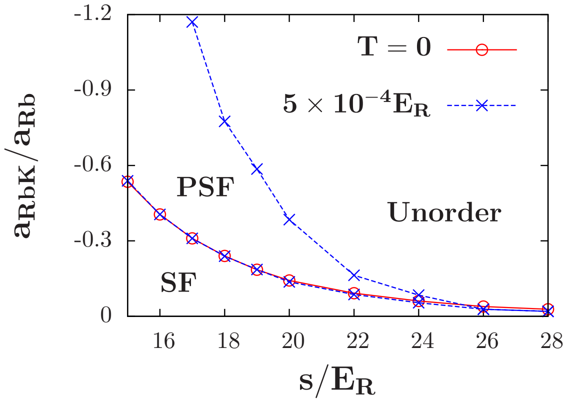

FIG. 5: Phase diagram for a mixture of ^{87}Rb and ^{41}K in a 3D cubic lattice as a function of lattice depth s and Rb-K scattering length. The total filling is $n = 1$ with $n_b = n_d = 0.5$.

Now we explore the phase diagram of ^{87}Rb and ^{41}K mixtures in a 3D cubic lattice and make predictions for ongoing experiments. Since the depth s of the optical lattice and the inter-species scattering length a_{RbK} are tunable with high accuracy, we show in Fig. 5 the phase diagram in the a_{RbK}-s plane for total filling $n = 1$ ($n_b = n_d$) at zero and finite temperatures. At zero temperature, two phases appear: superfluid and PSF. When the scattering length a_{RbK} is small, the system is in a superfluid phase for a shallow lattice. When the depth of the lattice is increased, the ratio of $t_{\mathrm{Rb}}/U_{\mathrm{Rb}}$ decreases, resulting in a strong suppression of first-order tunneling. The dominant process will then be hopping of composite pairs. At finite temperature, the PSF can be easily destroyed by thermal fluctuations which induce a second-order phase transition into a unordered phase with $\phi_{bd} = 0$. Since the PSF is formed via second-order tunneling and the corresponding energy scale is very small, for the parameters chosen here, this transition already occurs at a low temperature $T = 0.0005 E_R$. On the other hand we observe that at the same temperature the superfluid phase is very stable and the phase boundary between superfluid and PSF is almost unchanged.

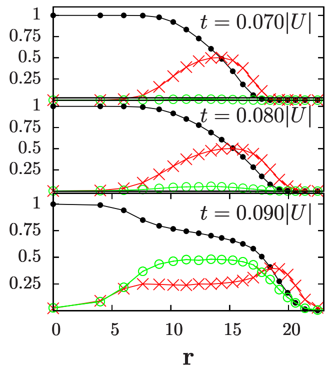

FIG. 6: Density distributions n_b (black line), order parameters ϕ_b (green line) and PSF correlator ϕ_{bd} (red line) vs. radial distance r for different hopping amplitudes at zero temperature in a 2D square lattice, obtained by RBDMFT. The interactions are $U_b = U_d = 2|U|$, hopping amplitudes $t = t_b = t_d$ and harmonic trap $V_0 = 0.0002|U|$.

C. Trapped Bose-Bose mixtures in 2D and 3D lattices

In this section, we simulate the two-component bosonic system in both 2D and 3D in the presence of a harmonic trap, as is relevant for most experiments. In particular, we investigate the stability of the PSF in the trapped system. Here we choose a 41×41 square lattice for the 2D case and a $41 \times 41 \times 41$ cubic lattice for the 3D case. In 2D, in our simulations we apply both RBDMFT and BDMFT+LDA, while in 3D we only use BDMFT+LDA due to its lower computational effort.

1. Balanced mixture

Fig. 6 shows the density distributions n_b, order parameter ϕ_b and correlator ϕ_{bd} for the PSF vs. radius r at different hopping amplitudes in a trapped 2D optical lattice. Since the PSF is stabilized only within a narrow region for the symmetric parameters (see Fig. 4), the harmonic trap should be very shallow and the hopping amplitudes need to be fine-tuned. Otherwise, the system will go through a phase transition directly from a Mott-insulating to a superfluid phase. Here we choose completely symmetric parameters: $t = t_b = t_d$ and $U_b = U_d = 2|U|$ with balanced filling for the two components. Therefore only one value for $n_{b,d}$ and $\phi_{b,d}$ respectively is shown in Fig. 6. We observe that a wedding-cake structure appears in the trapped system, and the coexistence of different phases sensitively depends on the hopping amplitudes. At lower hopping $t = 0.55|U|$, only two phases appear, and the corresponding phase transition is from a Mott insulator with total filling $n = 2$ to a PSF with total filling $0 < n < 2$ indicated by the non-vanishing value of ϕ_{bd} while $\phi_b = 0$. If we increase the hopping amplitudes, the first-order tunneling of single atoms will increase, which induces large density fluctuations in the system, leading to a phase transition from the PSF to the superfluid phase. We clearly observe this point from the middle panel of Fig. 6 where the superfluid phase starts to appear right in the middle of the PSF. With further increase of the tunneling amplitudes the superfluid dominates, as shown in the lower panel of Fig. 6, where the PSF completely disappears at $t = 0.7|U|$.

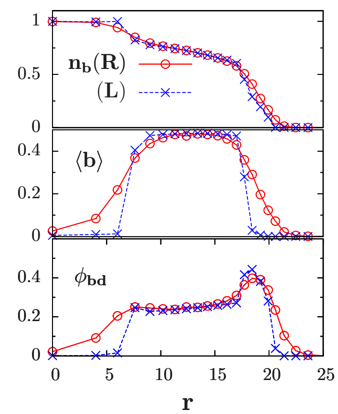

FIG. 7: Comparison between results from RBDMFT (R) and those from BDMFT+LDA (L) for a 2D square lattice. Density distributions n_b, order parameters ϕ_b and PSF correlator ϕ_{bd} vs. radial distance r at zero temperature in a 2D square lattice. The interactions are $U_b = U_d = 2|U|$ and the hopping amplitudes $t_b = t_d = 0.09|U|$ with a harmonic trap $V_0 = 0.0002|U|$.

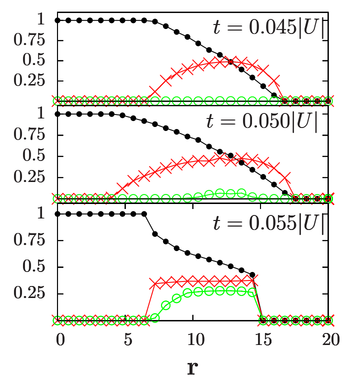

FIG. 8: Density distribution (black line), order parameter (green line) and PSF correlator ϕ_{bd} (red line) vs. radius r for different hopping amplitudes at zero temperature for a trapped 3D cubic lattice obtained within BDMFT+LDA. The interactions are $U_b = U_d = 12|U|$, hopping amplitudes $t = t_b = t_d$ and harmonic trap $V_0 = 0.0002|U|$.

Fig. 7 shows a comparison between the results of RBDMFT and those of BDMFT+LDA for a 2D square lattice. We observe good agreement between the two methods except close to the phase transition. In spite of the artificially sharp phase transition feature of LDA, the results of BDMFT+LDA are still reliable with sufficient accuracy in the regime away from the transition. We will therefore apply BDMFT+LDA to tackle the 3D case

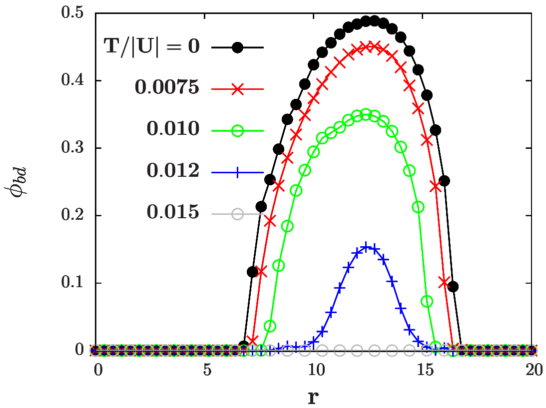

FIG. 9: Temperature dependence of the PSF correlator as a function of radius r for a 3D cubic lattice obtained within BDMFT+LDA. The interactions are $U_b = U_d = 12|U|$, hopping amplitudes $t_b = t_d = 0.045|U|$ and harmonic trap $V_0 = 0.0002|U|$.

due to its higher computational efficiency compared to RBDMFT.

Let us now investigate the stability of PSF in the 3D case in the presence of a harmonic trap. Results obtained within LDA are shown in Fig. 8. Here we choose completely symmetric parameters: $t = t_b = t_d$ and $U_b = U_d = 12|U|$ with balanced filling for the two components. And only one value for $n_{b,d}$ and $\phi_{b,d}$ is shown in Fig. 8, respectively. Compared to 2D, we observe a similar scenario of phase coexistence in the trapped 3D cubic lattice: at lower hopping amplitudes the Mott insulator and the PSF are coexisting, at intermediate hopping amplitudes, the superfluid phase appears due to increased density fluctuations, and at even larger hopping amplitudes, the PSF will disappear.

We are also interested in the effects of temperature on the PSF. Fig. 9 shows the correlator ϕ_{bd} for different temperatures. We observe that the PSF is sensitive to thermal fluctuations. At finite T, the PSF is reduced in favor of developing an unordered phase characterized by $\phi_{bd} = 0$. On the other hand, the density distribution is rather insensitive to small finite T.

2. *Imbalanced mixture*

As pointed out above, asymmetry in the hopping of the two species does not destroy the PSF. However, filling imbalance hinders the formation of the pairs [17, 34]. We will now study this effect in more detail. The imbalance, $N_b \neq N_d$, will be controlled by a nonzero chemical potential difference $\Delta\mu = \mu_b - \mu_d$ which can be viewed as an effective magnetic field. Results in 2D are shown in Fig. 10. Upon increasing $\Delta\mu$, the PSF will cease to exist and be replaced by a superfluid phase, since the chemical potential difference eventually exceeds the pairing gap for the PSF, allowing unpaired excess atoms to enter the PSF region. We find that even small population

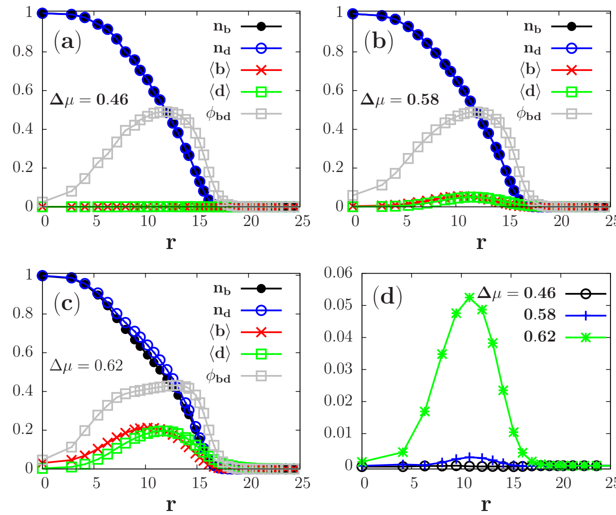

FIG. 10: Density distributions $n_{b,d}$, order parameters $\langle b \rangle$, $\langle d \rangle$ and PSF correlator ϕ_{bd} vs. radial distance r for different $\Delta\mu$ at zero temperature in a trapped 2D cubic lattice, obtained within BDMFT+LDA. Panel (d) shows the filling difference $(n_b - n_d)$ vs. radius r. The interactions are $U_b = U_d = 12|U|$, hopping amplitudes $t_b = t_d = 0.05|U|$, $(\mu_b + \mu_d)/2 = 0.48$ and harmonic trap $V_0 = 0.00015|U|$.

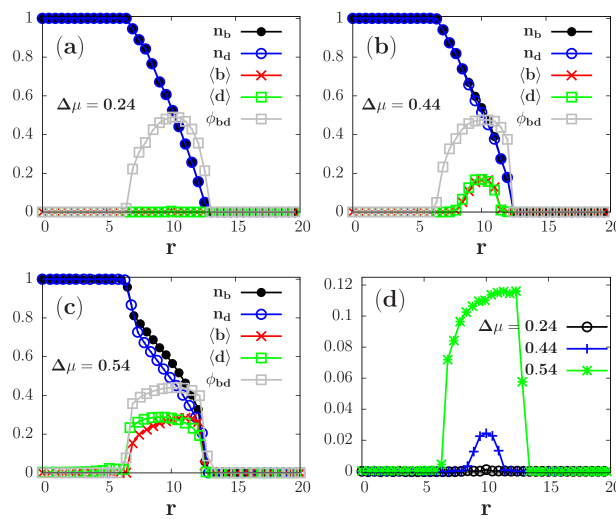

FIG. 11: Density distributions $n_{b,d}$, order parameters $\langle b \rangle$, $\langle d \rangle$ and PSF correlator ϕ_{bd} vs. radius r for different $\Delta\mu$ at zero temperature in a trapped 3D cubic lattice, obtained within BDMFT+LDA. Panel (d) shows the filling imbalance $(n_b - n_d)$ vs. radius r. The interactions are $U_b = U_d = 12|U|$, hopping amplitudes $t_b = t_d = 0.04|U|$, $(\mu_b + \mu_d)/2 = 0.47$ and harmonic trap $V_0 = 0.0003|U|$.

imbalance can destroy the PSF in a trapped system. On the other hand, as shown in Fig. 10, the density distribution is almost unchanged for similar parameters. When increasing the imbalance parameter $\Delta\mu$ further, the PSF will disappear already for a small population imbalance, since the particles form a conventional superfluid. Here we do not find any phase separation.

Finally we discuss the influence of population imbalance on the PSF in a trapped 3D cubic lattice using BDMFT+LDA. From our results shown in Fig. 11 we conclude that here the physics is qualitatively similar to the 2D case.

IV. SUMMARY

We have investigated low-temperature properties of Bose-Bose mixtures with attractive inter-species interaction both in 2D and 3D optical lattices by means of BDMFT/RBDMFT. In particular, we found that the pair-superfluid is stable also for asymmetric hopping of the two species. We obtained the critical temperature of the PSF which we found to be of the same order as that of the XY-ferromagnet in the corresponding system with repulsive interactions of equal magnitude. We have confirmed the stability of the PSF in a balanced Bose-Bose-mixture in the presence of the harmonic trap both in 2D and 3D. On the other hand, we found that even a small population imbalance can destroy the PSF. This novel PSF quantum phase can be detected in future experiments via the momentum distribution of pairs, which shows signatures of the pair condensate [17, 34].

Acknowledgments

We acknowledge useful discussions with M. Reza Bakhtiari. This work was supported by the China Scholarship Fund (Y.L) and the Deutsche Forschungsgemeinschaft via SFB-TR/49 and the DIP project HO 2407/5-1.

---

[1] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).
[2] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch and I. Bloch, Nature (London) **415**, 39 (2002).
[3] J. Catani, L. Desarlo, G. Baronitini, F. Minardi and M. Inguscio, Phys. Rev. A **77**, 011603(R) (2008).
[4] D. M. Weld, P. Medley, H. Miyake, D. Hucul, D. E. Pritchard, and Wolfgang Ketterle, Phys. Rev. Lett. **103**, 245301 (2009).
[5] P. Medley, D. M. Weld, H. Miyake, D. E. Pritchard and W. Ketterle, Phys. Rev. Lett. **106**, 195301 (2011).
[6] B. Gadway, D. Pertot, R. Reimann and D. Schneble, Phys Rev. Lett. **105**, 045303 (2010).
[7] M. Lewenstein and A. Sanpera, Science **319**, 292 (2008).
[8] A. J. Daley, J. M. Taylor, S. Diehl, M. Baranov and P. Zoller, Phys. Rev. Lett. **102**, 040402 (2009).
[9] Y. Lee and M. Yang, Phys. Rev. A **81**, 061604 (2010).
[10] L. Bonnes and S. Wessel, Phys. Rev. Lett. **106**, 185302 (2011).
[11] Y. Chen, K. Ng and M. Yang, Phys. Rev. B **84**, 092503 (2011).
[12] A. Kuklov, N. Prokof'ev and B. V. Svistunov, Phys. Rev. Lett. **92**, 030403 (2004); A. Kuklov, N. Prokof'ev and B. Svistunov, Phys. Rev. Lett. **92**, 050402 (2004).
[13] A. Arguelles and L. Santos, Phys. Rev. A **75**, 053613 (2007).
[14] S. Guertler, M. Troyer and F. Zhang, Phys. Rev. B **77**, 184505 (2008).
[15] L. Mathey, I. Danshita and C. W. Clark, Phys. Rev. A **79**, 011602 (2009).
[16] A. Hu, L. Mathey, I. Danshita, E. Tiesinga, C. J. Willians and C. W. Clark, Phys. Rev. A **80**, 023619 (2009).
[17] C. Menotti and S. Stringari, Phys. Rev. A **81**, 045604 (2010).
[18] M. Iskin, Phys. Rev. A **82**, 033630 (2010); M. Iskin, J. Phys. A **44**, 275301 (2011).
[19] P. Cheng and M. Yang, Phys. Rev. B **82**, 180510(R) (2010).
[20] A. B. Kuklov and B. V. Svistunov, Phys. Rev. Lett. **90**, 100401 (2003).
[21] E. Altman, W. Hofstetter, E. Demler and M. D. Lukin, New. J. Phys. **5**, 113 (2003).
[22] S. Foelling, S. Trotzky, P. Cheinet, M. Feld, R. Saers, A. Widera, T. Mueller and I. Bloch, Nature (London) **448**, 1029 (2007).
[23] S. Trotzky, P. Cheinet, S. Folling, M. Field, U. Schnorrberger, A.M. Rey, A. Polovnikov, E.A. Demler, M.D. Lukin and I. Bloch, Science **319**, 295 (2008).
[24] U. Shrestha, Phys. Rev. A **82**, 041603 (2010).
[25] W. S. Bakr, J. I. Gillen, A. Peng, S. Foelling and M. Greiner, Nature (London) **462**, 74 (2009).
[26] J. F. Sherson, C. Weitenberg, M. Endres, M. Cheneau, I. Bloch and S. Kuhr, Nature (London) **467**, 68 (2010).
[27] K. Byczuk and D. Vollhardt, Phys. Rev. B **77**, 235106 (2008); A. Hubener, M. Snoek and W. Hofstetter, Phys. Rev. B **80**, 245109 (2009); W.-J. Hu and N.-H. Tong, Phys. Rev. B **80**, 245110 (2009); P. Anders, E. Gull, L. Pollet, M. Troyer and P. Werner, Phys. Rev. Lett. **105**, 096402 (2010).
[28] Y.-Q. Li, M. R. Bakhtiari, L. He and W. Hofstetter, Phys. Rev. B **84**, 144411 (2011); Y.-Q. Li, M. R. Bakhtiari, L. He and W. Hofstetter, Phys. Rev. A **85**, 023624 (2012).
[29] G. Thalhammer, G. Barontini, L. De Sarlo, J. Catani, F. Minardi and M. Inguscio, Phys. Rev. Lett. **100**, 210402 (2008).
[30] A. Widera, O. Mandel, M. Greiner, S. Kreim, T. W. Hänsch, and I. Bloch, Phys. Rev. Lett. **92**, 160406 (2004).
[31] M. Caffarel and W. Krauth, Phys. Rev. Lett. **72**, 1545 (1994).
[32] D. C. McKay and B. DeMarco, Rep. Prog. Phys. **74**, 054401 (2011).
[33] C. A. Regal and D. S. Jin, Phys. Rev. Lett. **90**, 230404 (2003).
[34] A. Hu, L. Mathey, C. J. Williams and C. W. Clark, Phys. Rev. A **81**, 063602 (2010).